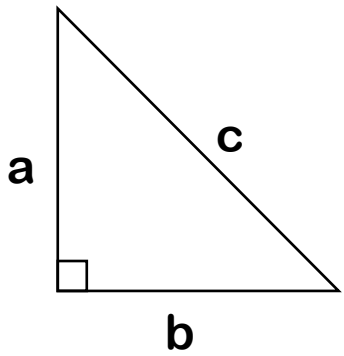
Right Angle Trigonometry

Trigonometry is the study of triangles in mathematics. We study angles and relationships with other angles and sides in a triangle.

In physics and engineering, we use this branch of math to help us calculate forces in structures, velocity of speeding objects, acceleration of rockets, and so much more!

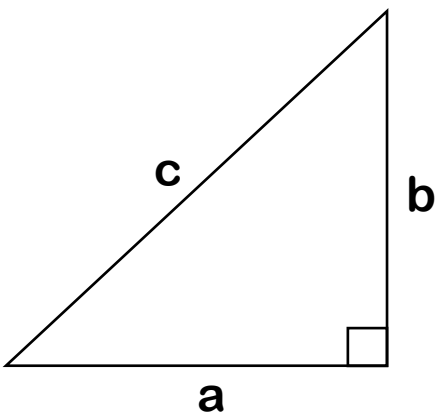
Parts of a Right Triangle



Sides "a" and "b" are <u>legs</u>. They form a right angle at their intersection.

Side "c" is the <u>hypotenuse</u>. This side is always opposite of the right angle.

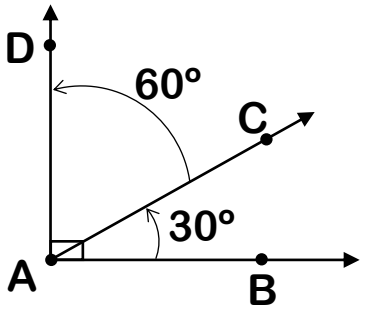
The hypotenuse is always the longest side.



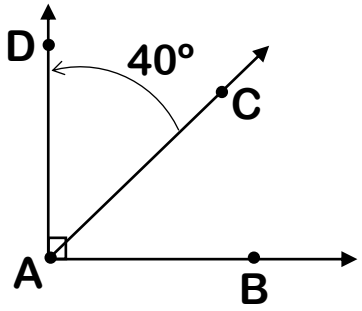
Types of Angles

Complementary angles always sum to 90°.

$\angle$DAC and $\angle$CAB are complementary.

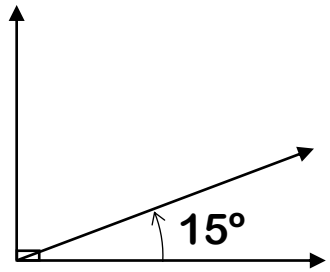


Find the missing angle:



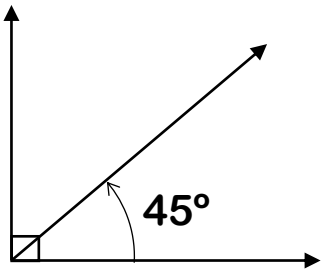
$90° - 40° = 50°$

$\angle CAB = 50°$



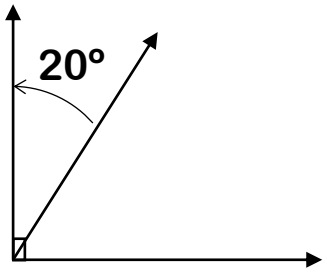
$90° - 15° = 75°$

75°



$90° - 45° = 45°$

45°

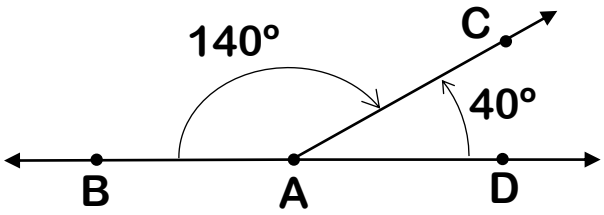


$90° - 20° = 70°$

70°

Supplementary angles always sum to 180°.

$\angle BAC = 140°$ $\angle CAD = 40°$

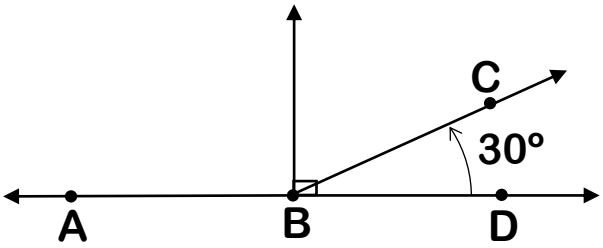


$\angle DAB = \angle BAC + \angle CAD$

$\angle DAB = 140 + 40°$

$= 180°$

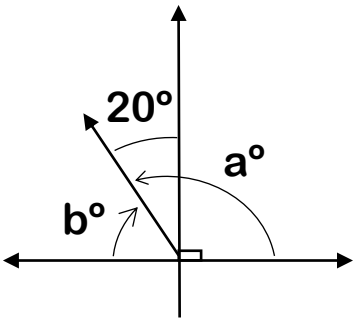
Find the missing angle:



$180° - \angle CBD$

$180° - 30° = 150°$

$\angle ABC = 150°$

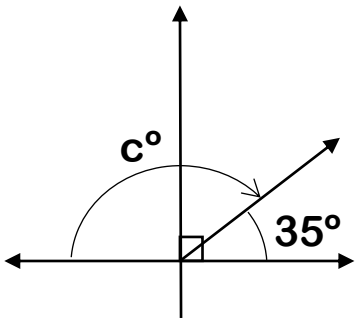


90+20=110

180-110=70

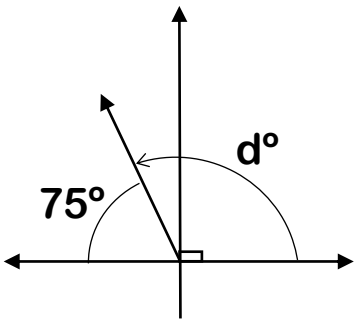
a° = 110 °

b° = 70 °



180-35=145

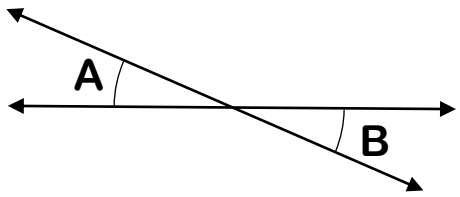
c° = 145 °



180-75=105

d° = 105 °

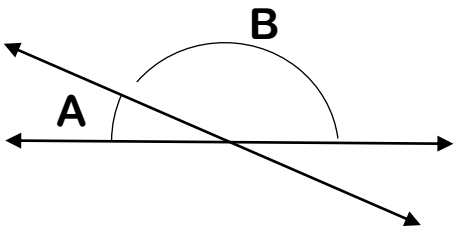
Angle Pairs



<u>Vertical</u> angles are opposite angles formed by intersecting lines.

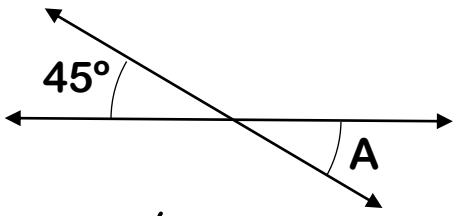
$\angle A = \angle B$

<u>Adjacent</u> angles are two angles that have a common side <u>and</u> vertex.

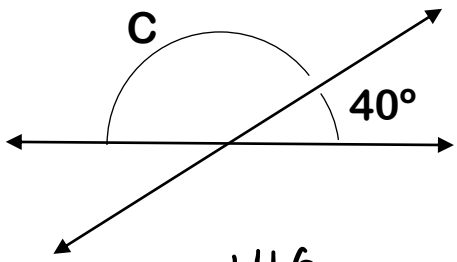


$\angle A + \angle B = 180°$

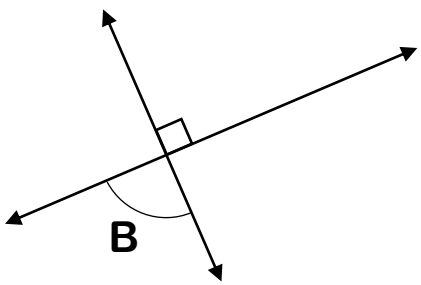
Find the missing angle:



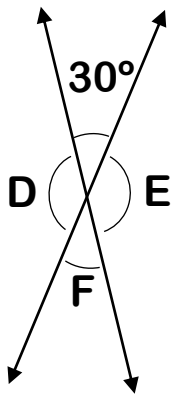
$\angle A = 45°$



$\angle C = 140°$



$\angle B = 90°$



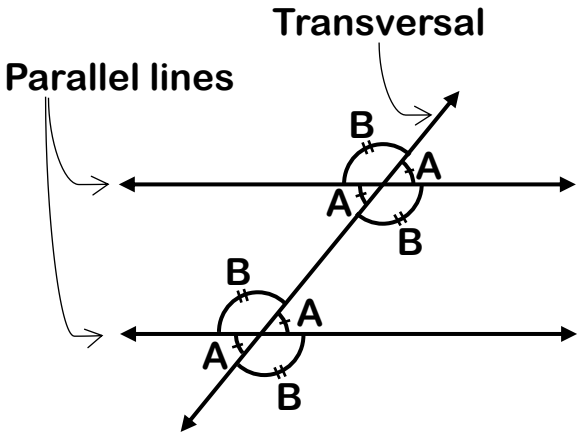
$\angle D = 150°$

$\angle E = 150°$

$\angle F = 30°$

Parallel Lines & Transversals

These parallel lines are intersected (cut) by a transversal, forming 8 different angles.

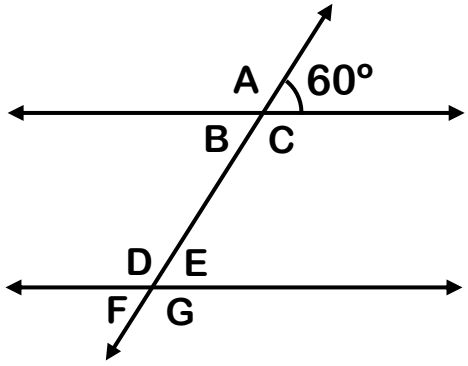


∡ A are all congruent.

∡ B are all congruent.

∡ A is supplementary to ∡ B .

Label all 8 angles:



∡ A = 120 °

∡ B = 60 °

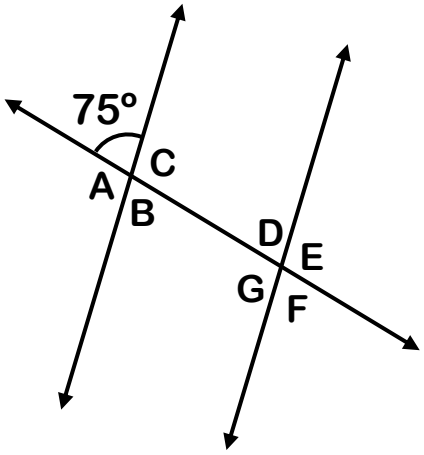
∡ C = 120 °

∡ D = 120 °

∡ E = 60 °

∡ F = 60 °

∡ G = 120 °



∡ A = 105 °

∡ B = 75 °

∡ C = 105 °

∡ D = 75 °

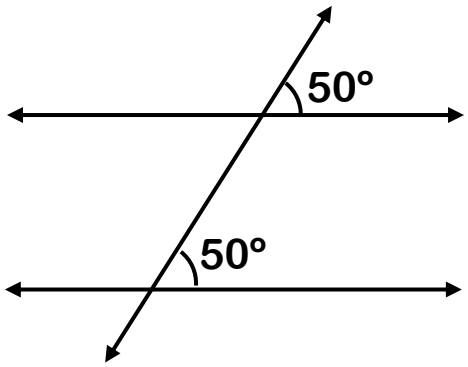
∡ E = 105 °

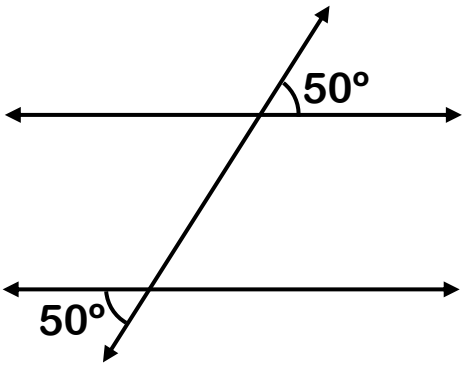
∡ F = 75 °

∡ G = 105 °

Types of Angle Pairs

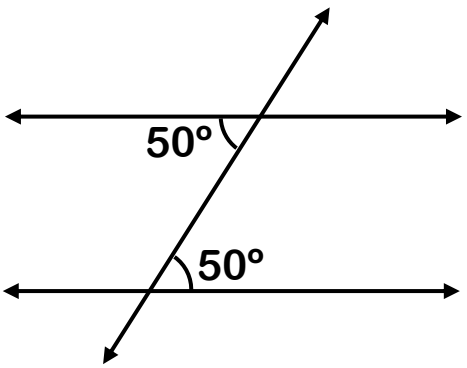
Corresponding angles are in the same relative position and are all congruent.

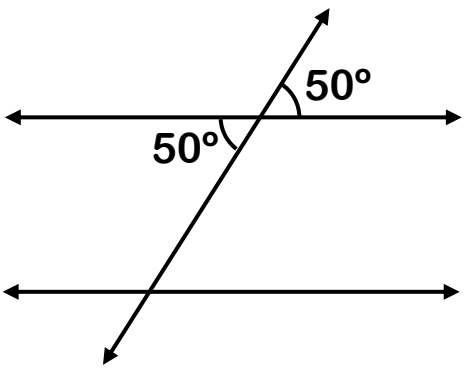




Alternating exterior angles are outside the parallel lines and on opposite sides of the transversal.

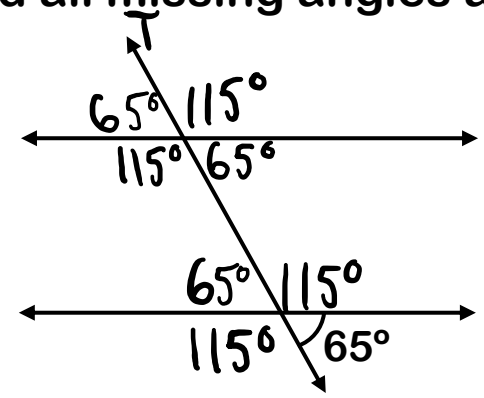
Alternating interior angles are on the inside of the parallel lines and on opposite sides of the transversals.

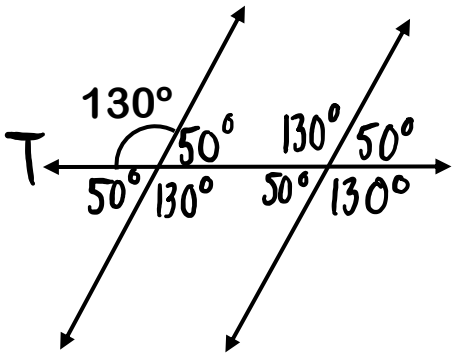




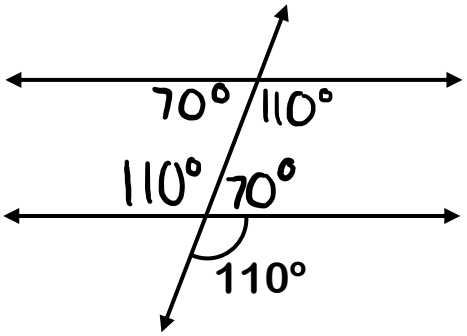
Vertical angles are formed by intersecting lines and are on opposite sides.

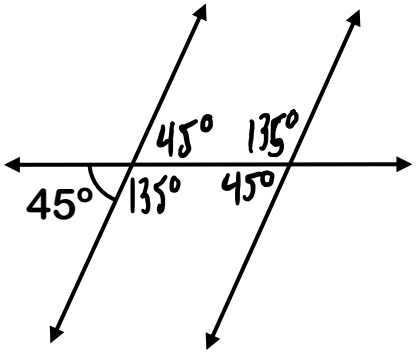
Find all missing angles and label the transversal:



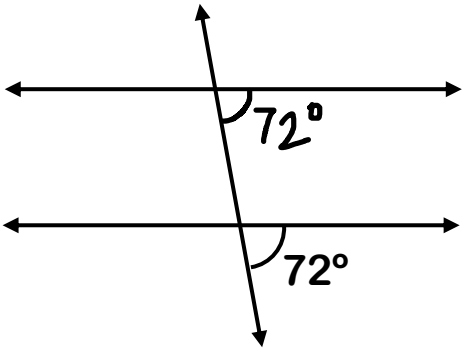


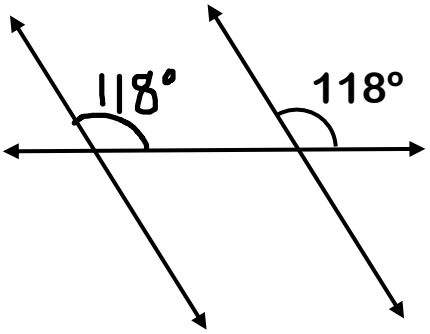
Label only alternating interior angles:



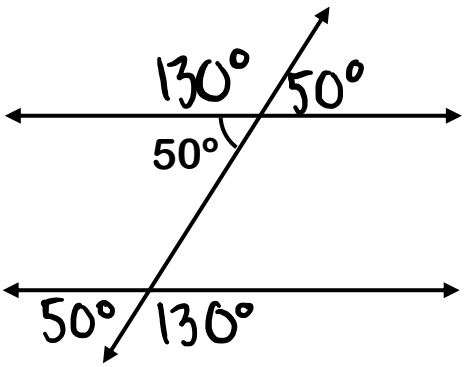


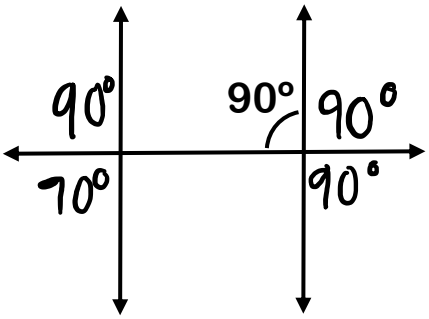
Label only the corresponding angle:





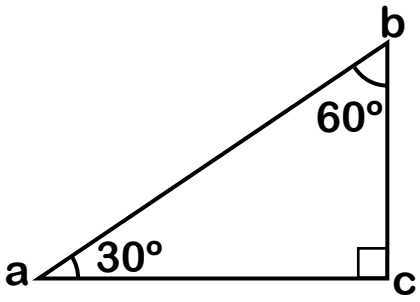
Label only the alternating exterior angles:





Angles of a Triangle

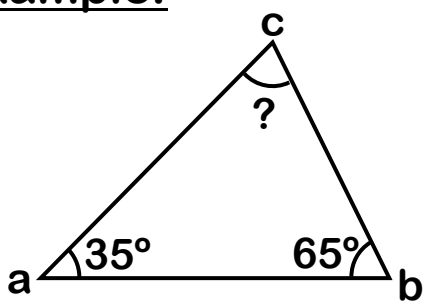
The sum of all angles in a triangle is always 180°.



$\angle a + \angle b + \angle c = 180°$

$30° + 60° + 90° = 180°$

Example:



$\angle a + \angle b + \angle c = 180°$

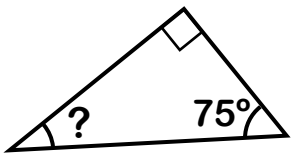
$\angle 35° + \angle 65° + \angle c = 180°$

$\angle 100° + \angle c = 180°$

$\angle c = 180° - 100°$

$\angle c = 80°$

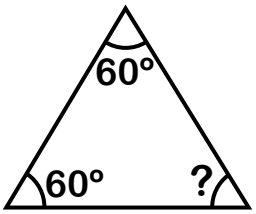
Find the missing angles:



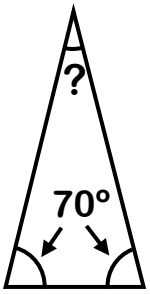
180-90=90

90-75=15

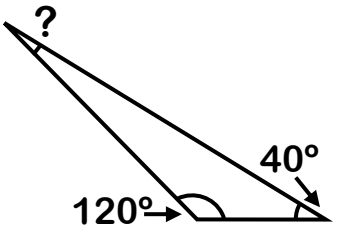
15°



60°



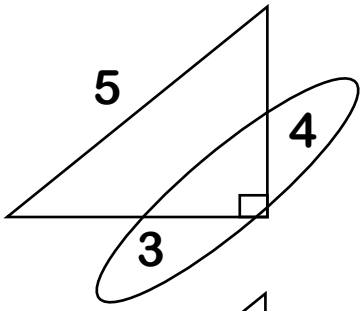
40°



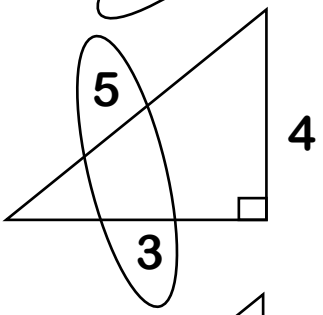
20°

Triangle Constraints

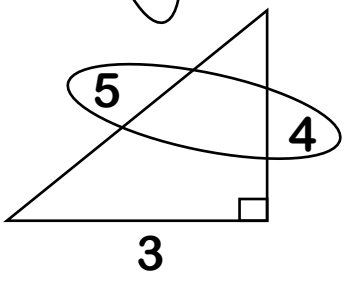
The sum of any two sides must be greater than the third.



$3 + 4 > 5$



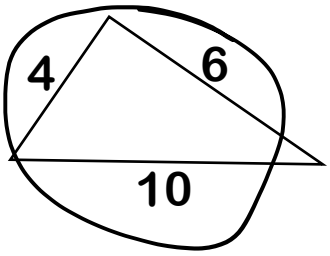
$5 + 3 > 4$

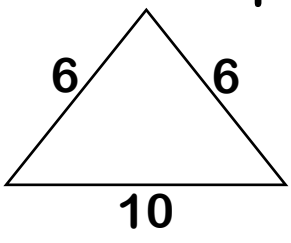


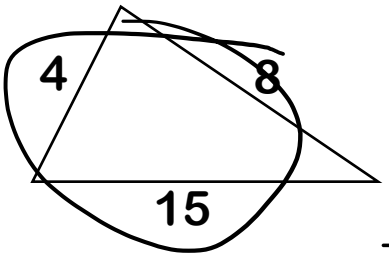
$5 + 4 > 3$

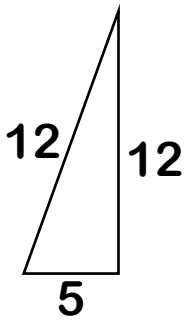
One Unique Triangle:
3 side lengths that meet these requirements will always make 1 unique triangle

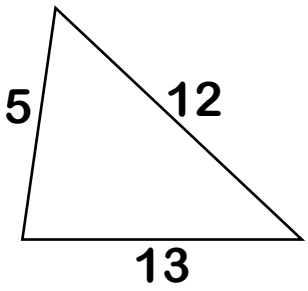
Which triangles are impossible?

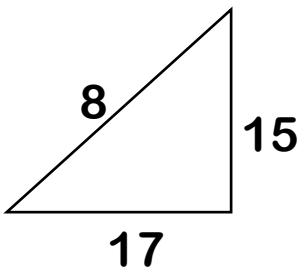


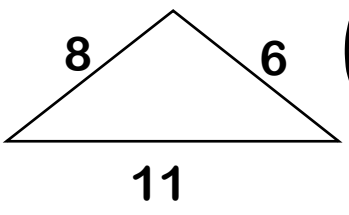


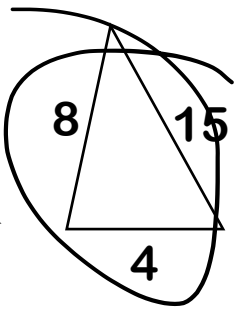






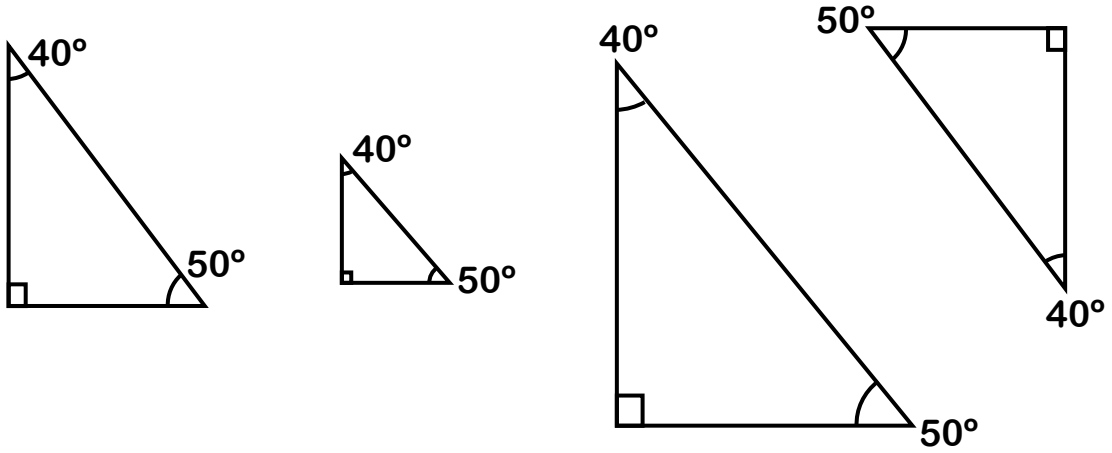






Triangle Constraints

More than one triangle can be made from knowing only the angles of a triangle. These are called <u>similar triangles</u>, or congruent triangles



In order to make a congruent triangle unique, you must constrain (specify) at least <u>one</u> side length.

How many triangles are possible?

a) side lengths: 4, 2, 6 (0)

b) side lengths: 5, 12, 13 (1)

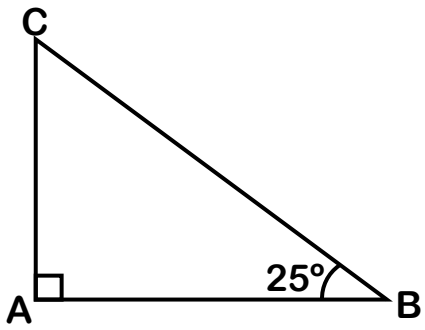
c) side lengths: 3, 4, 5 (1)

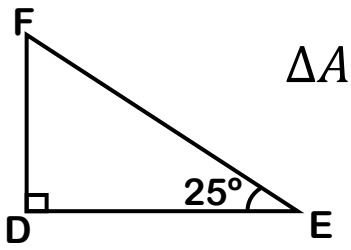
d) ∡ 30°, 60°, 90° (∞)

e) ∡ 45°, 55°, 95° (0)

f) ∡ 30°, 45°, and hypotenuse=7 (1)

Similar triangles: If two triangles have two pairs of corresponding angles that are congruent, then the triangles are similar

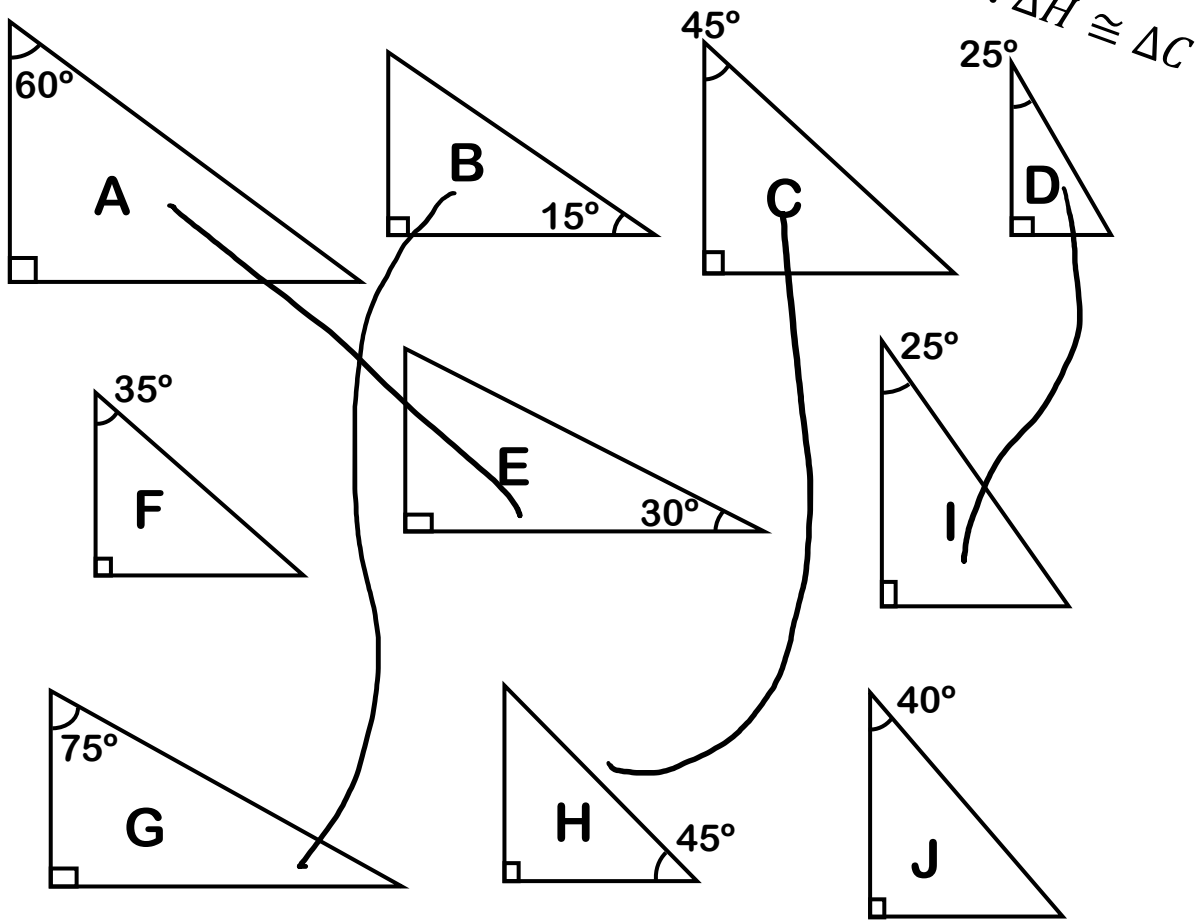




$\Delta ABC \cong \Delta DEF$

Which triangles are similar?

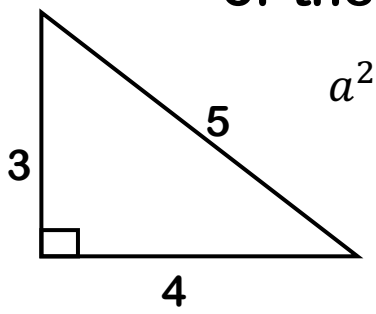
Example: $\Delta H \cong \Delta C$



Pythagorean Theorem

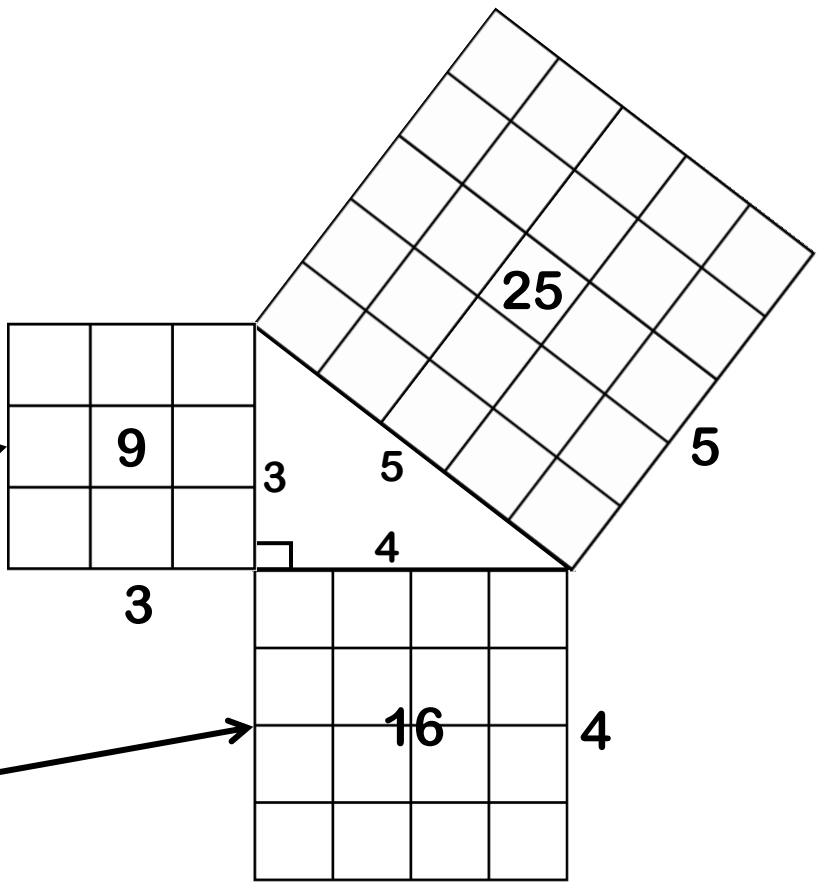
only for right Δ's

In a right triangle, the sum of the squares of the shorter sides equals the squares of the hypotenuse.



$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25 \checkmark$$



The areas of the two smaller squares always equal the area of the largest square.

This means that is you check to see if $a^2 + b^2 = c^2$ and it's true, then you know it's a right triangle.

Pythagorean Converse

Which triangles are right triangles?

side lengths:

① **5, 12, 13** $5^2 + 12^2 = 13^2$

$25 + 144 = 169$ ✓

$169 = 169$

② **3, 4, 7** $3^2 + 4^2 = 7^2$

$9 + 16 = 49$ ✗

③ **8, 15, 17** $8^2 + 15^2 = 17^2$

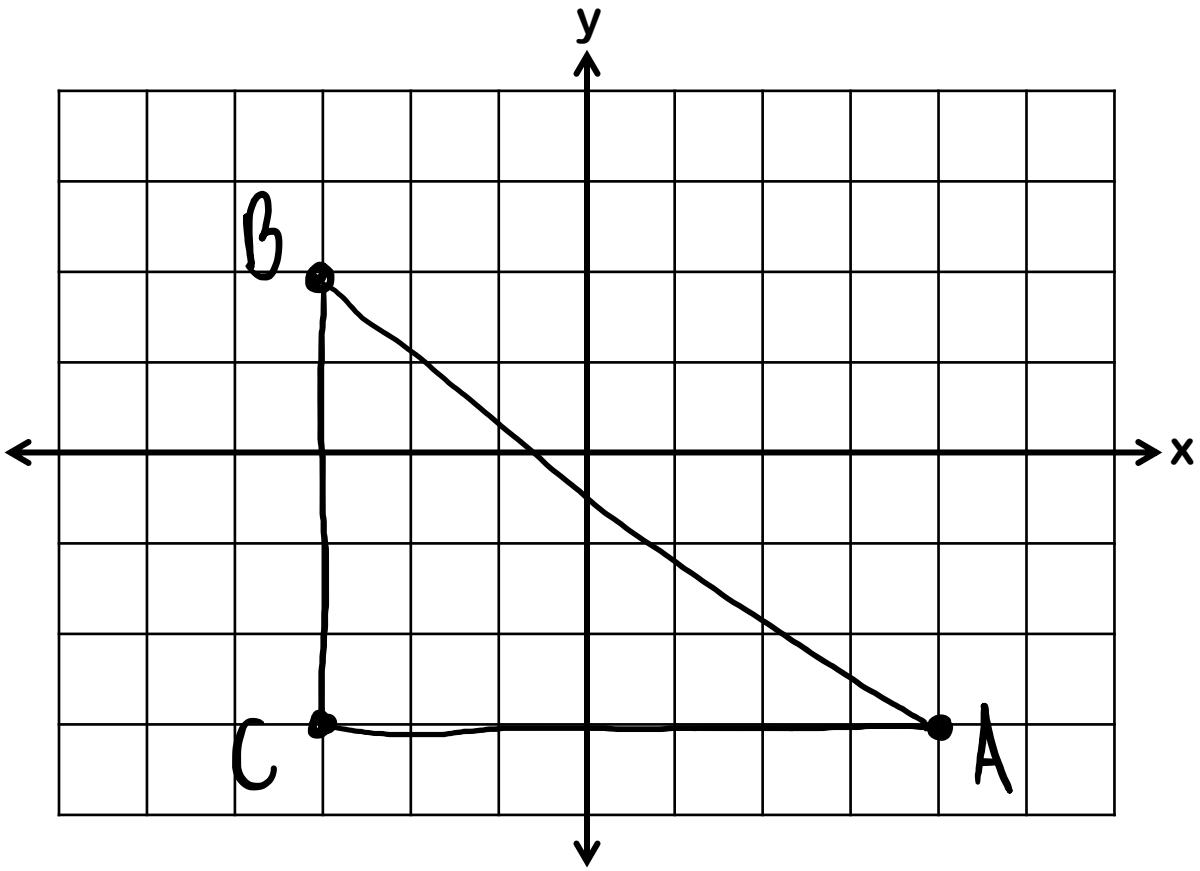
$64 + 225 = 289$ ✓

$289 = 289$

Using a ruler,
Draw one of the right triangles from above using either cm or mm on your ruler (whichever is best).

answers will vary

Graphing Triangles



① **Plot the points:** $A = (4, -3)$
$B = (-3, 2)$
$C = (-3, -3)$

② **Connect the dots to form a triangle**

③ **Use your protractor to measure each angle:**

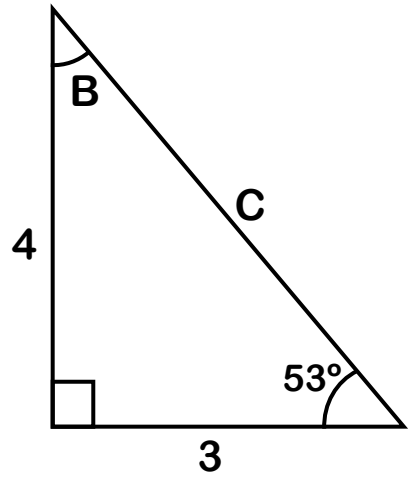
$\measuredangle A = 36.9^\circ$
$\measuredangle B = 53.1^\circ$
$\measuredangle C = 90$

④ **Use your ruler to measure the hypotenuse**

4 inches

⑤ **Compare the measured hypotenuse with using the Pythagorean Theorem to find the diagonal distance between points A and B**

Finding sides and angles of Right triangles:



Use your understanding of angles and the Pythagorean theorem to find all the missing sides and angles for each triangle

$\measuredangle B = 180° - 90° - 53°$

$\underline{\measuredangle \boldsymbol{B = 37°}}$

$c^2 = 4^2 + 3^2$

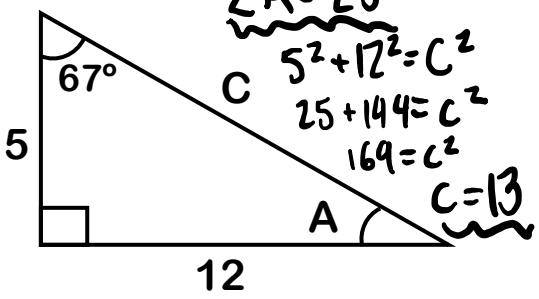
$c^2 = 16 + 9$

$c^2 = 25$

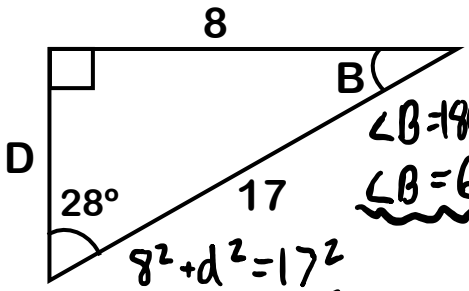
$\sqrt{c^2} = \sqrt{25}$

$\underline{\boldsymbol{c = 5}}$

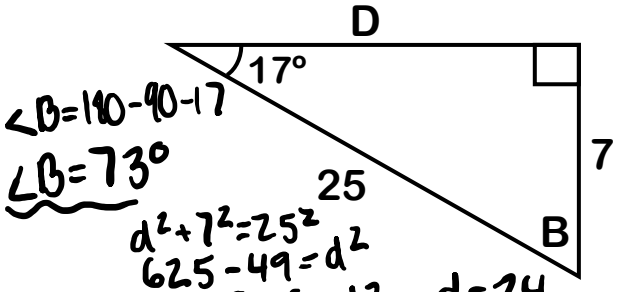
Now you try:



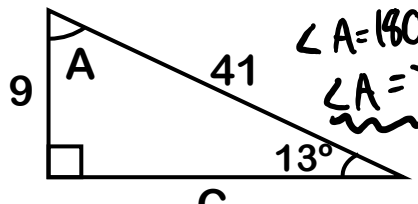
∠A=180-90-67
∠A=23°
$5^2+12^2=C^2$
$25+144=C^2$
$169=C^2$
C=13



∠B=180-90-28
∠B=62°
$8^2+d^2=17^2$
$289-64=d^2$
$225=d^2$
d=15



∠B=180-90-17
∠B=73°
$d^2+7^2=25^2$
$625-49=d^2$
$576=d^2$
d=24



∠A=180-90-13
∠A=77°
$9^2+c^2=41^2$
$81+c^2=1681$
$c^2=1600$
c=40

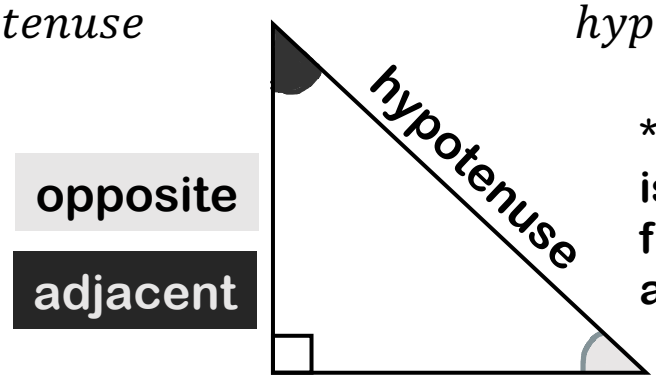
Trigonometry Ratios

Sine (sin) is the ratio of

$$\frac{opposite\ side}{hypotenuse}$$

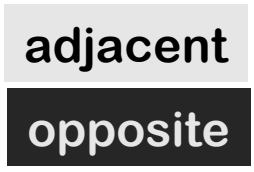
Cosine (cos) is the ratio of

$$\frac{adjacent\ side}{hypotenuse}$$

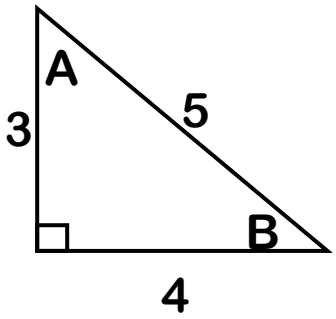


*The hypotenuse is always across from the right angle.

*The opposite and adjacent sides depend on the angle



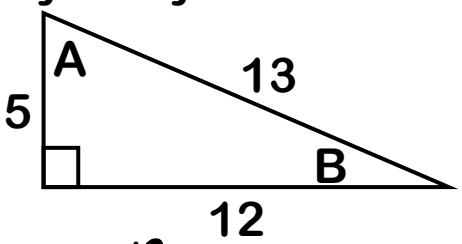
Example:



$$\sin(A) = \frac{4}{5} \qquad \cos(A) = \frac{3}{5}$$

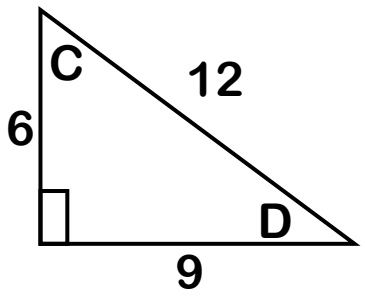
$$\sin(B) = \frac{3}{5} \qquad \cos(B) = \frac{4}{5}$$

Now you try:



$$\sin(A) = \frac{12}{13} \qquad \cos(A) = \frac{5}{13}$$

$$\sin(B) = \frac{5}{13} \qquad \cos(B) = \frac{12}{13}$$



$$\sin(C) = \frac{9}{12} \qquad \cos(C) = \frac{6}{12}$$

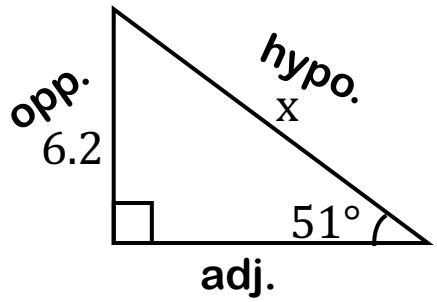
$$\sin(D) = \frac{6}{12} \qquad \cos(D) = \frac{9}{12}$$

Using ratios to find the missing side/angle

Steps to find a missing side

1. Label the sides (opposite, adjacent, hypotenuse)
2. Identify which trig ratio to use (sin, cos)
3. Set up an equation
4. Solve (using a calculator)

Example:



$$\sin(\theta) = \frac{opposite}{hypotenuse}$$

$$\sin(51) = \frac{6.2}{x}$$

$$6.2 \div \sin(51) = x$$

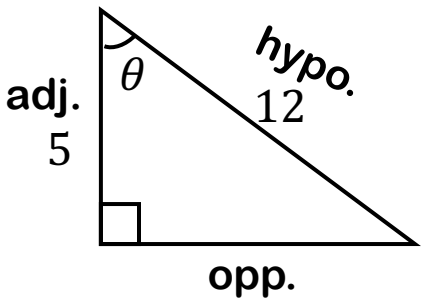
$$\underline{x = 8}$$

Steps to find a missing angle

1. Label the sides (opposite, adjacent, hypotenuse)
2. Identify which trig ratio to use (sin, cos)
3. Set up an equation
4. Use the inverse trig ratio ($\sin^{-1}$, $\cos^{-1}$) to solve

*Theta (θ) is the variable for an angle

Example:



$$cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$cos(\theta) = \frac{5}{12}$$

$$cos^{-1}\left(\frac{5}{12}\right) = x$$

$$\underline{x = 65°}$$

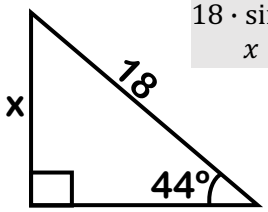
Sines- Math Lib

Wearing a (1) top hat and (2) monocle, the (3) dog was (4) singing the (5) national anthem and (6) dancing on top of a (7) stack of pancakes to impress (8) their new friends.

Find the missing sides to fill in the blanks (use a calculator):

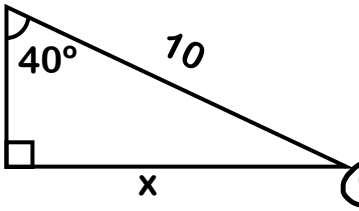
1

$$\sin(44°) = \frac{x}{18}$$
$$18 \cdot \sin(44°) = x$$
$$x = 12.5$$



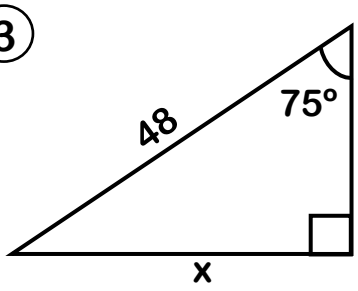
12.5→ top hat
14→ dress
18→ scarf
16.7 → blazer

2



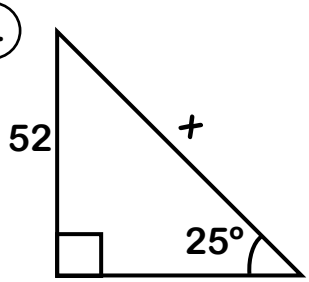
15.5→ earrings
13→ necklace
6.4→ monocle
7.7→ hair wig

3



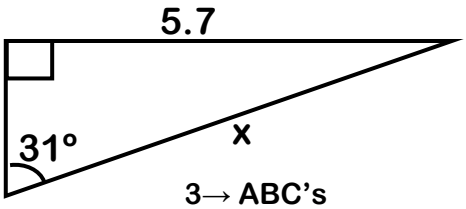
18.5→ mouse
12.4→ cat
50→ lizard
46.4→ dog

4



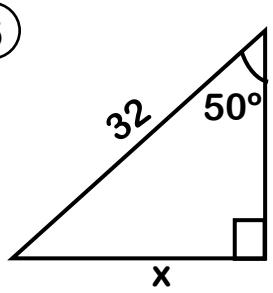
22→ yelling
123→ singing
47→ whispering
50.7→ smelling

5



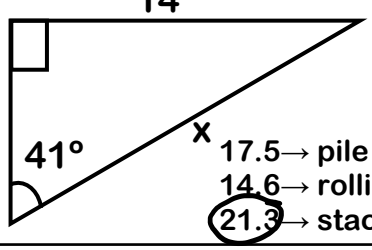
3→ ABC's
15→ Gettysburg Address
11→ National Anthem

6



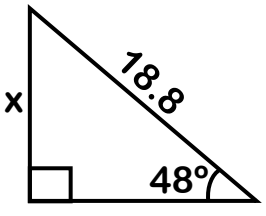
24.5→ dancing
20→ jumping
18→ twirling
41.7→ skating

7



17.5→ pile of pillows
14.6→ rolling skateboard
21.3→ stack of pancakes

8

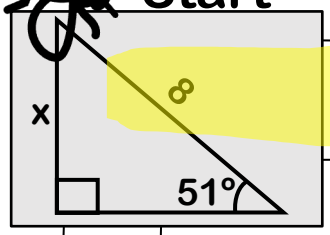


14→ their new friends
15→ the president
18→ the toddlers

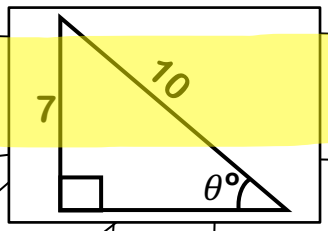
Sines- Maze

Determine the missing length of the right triangle described in each box and follow the answer to the next box.
The right answers will lead the bee to the flowers!

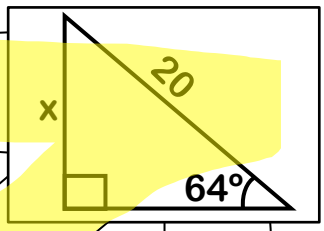
Start



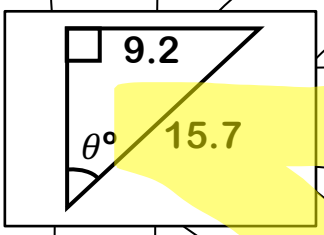
6.2



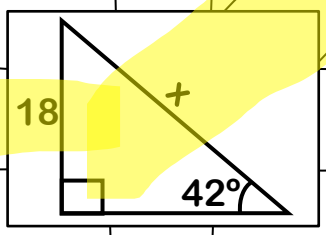
44



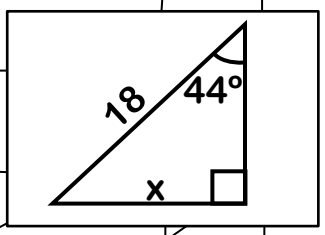
8 | 17 | 24 | 18 | 16



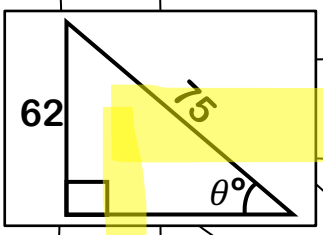
27



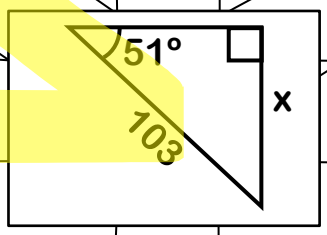
18



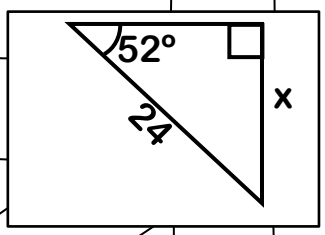
47 | 36 | 16 | 52 | 18



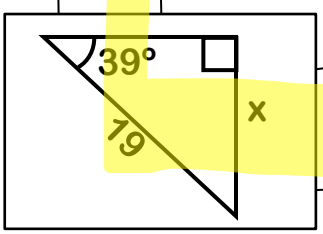
80



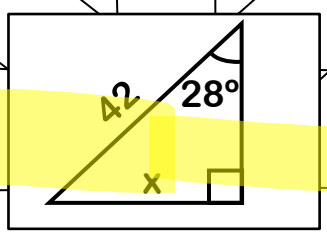
41



56 | 50 | 45 | 23 | 12

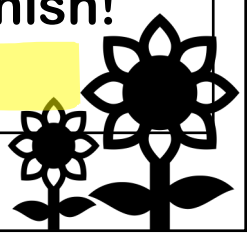


12



20

Finish!



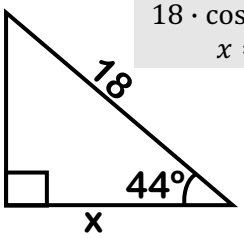
Cosines- Math Lib

The (1) teacher, wearing a (2) cape made of (3) newspaper and holding a (4) wand made of (5) rulers, (6) wrote using (7) glow in the dark paint while speaking in (8) rhymes.

Find the missing sides to fill in the blanks (use a calculator):

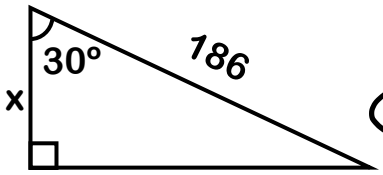
1

$$\cos(44°) = \frac{x}{18}$$
$$18 \cdot \cos(44°) = x$$
$$x = 13$$



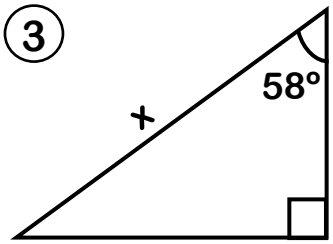
12.5→ driver
13→ teacher
18→ pilot
16.7 → tutor

2



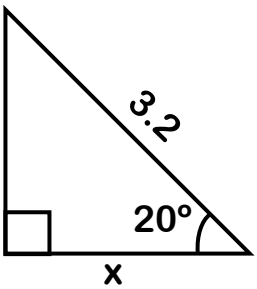
161.1→ cape
113→ necklace
64→ hat
177→ scarf

3



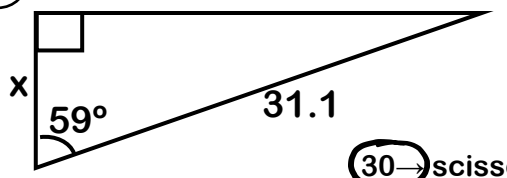
18.5→ yarn
56.6→ newspaper
52→ fur
44.4→ rubber

4



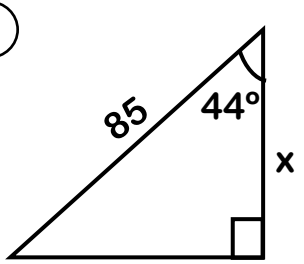
3→ wand
2→ baton
1.7→ pencil
0.6→ hairbrush

5



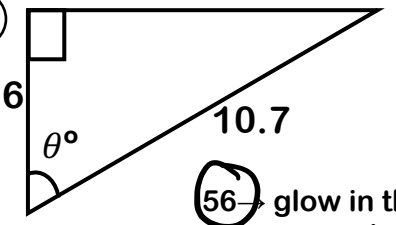
30→ scissors
15.9→ markers
31.1→ rulers

6



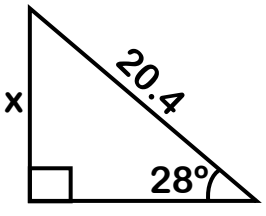
54.5→ danced
61.1→ wrote
82 → sang
49.7→ skipped

7



56→ glow in the dark
47→ neon pink
22→ sparkly

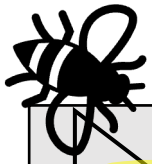
8



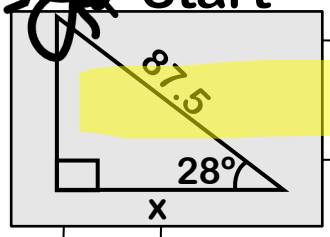
14→ French
15→ Shakespearean
18→ Rhymes

Cosines- Maze

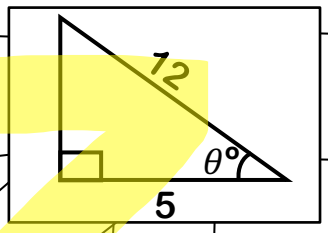
Determine the missing length of the right triangle described in each box and follow the answer to the next box.
The right answers will lead the bee to the flowers!



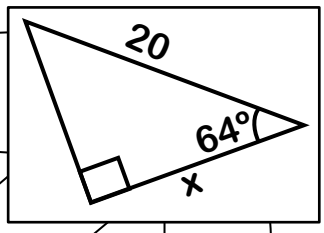
Start



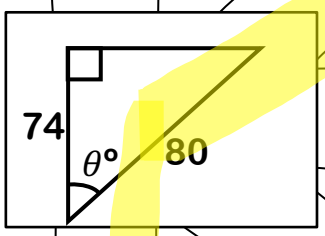
77.3



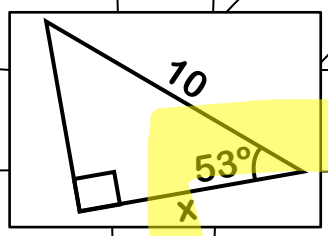
44



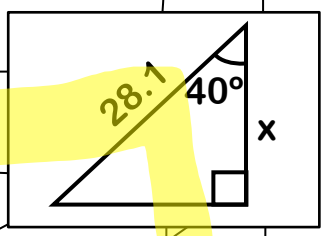
8 | 65 | 24 | 18 | 16



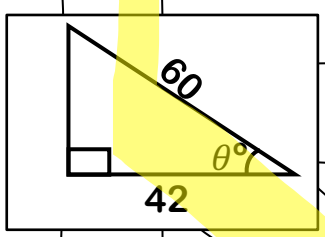
27



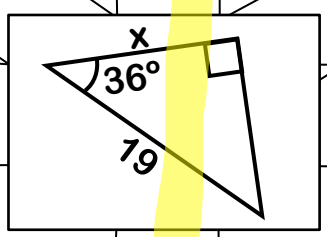
6



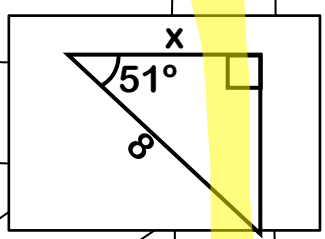
22 | 36 | 15.4 | 52 | 21.4



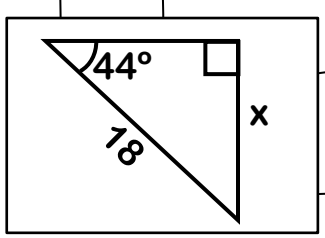
80



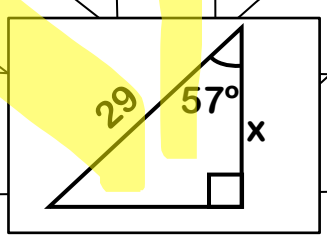
41



56 | 46 | 16 | 23 | 5

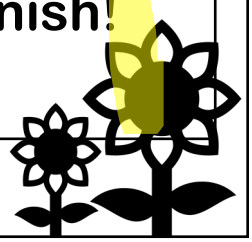


30



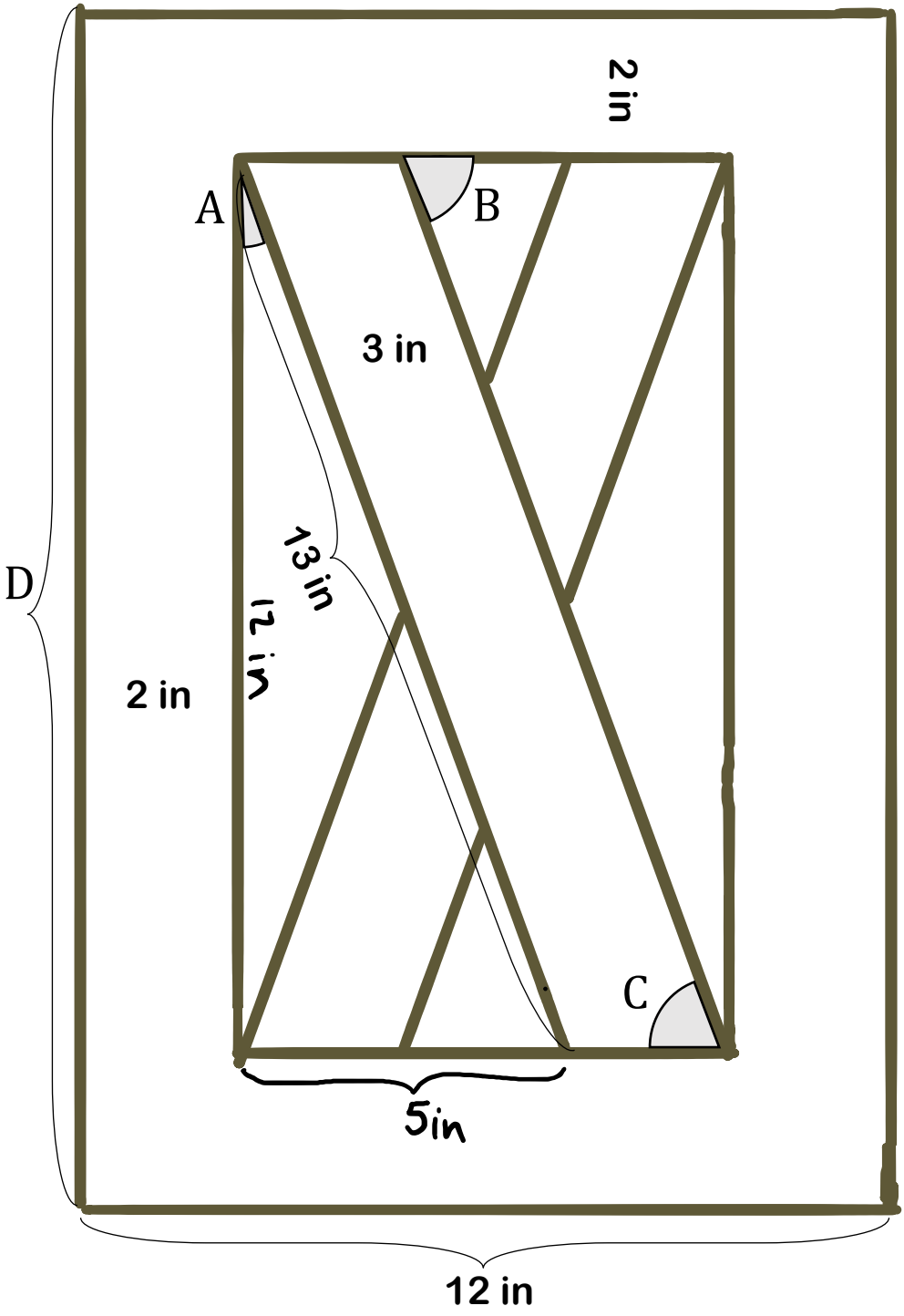
19

Finish!



Real World Applications for Right Triangles
Practice Test

Find the missing measurements of this door:



$\measuredangle A =$ 22.6°

$\measuredangle B =$ 67.4°

$\measuredangle C =$ 67.4°

$D =$ 16 in

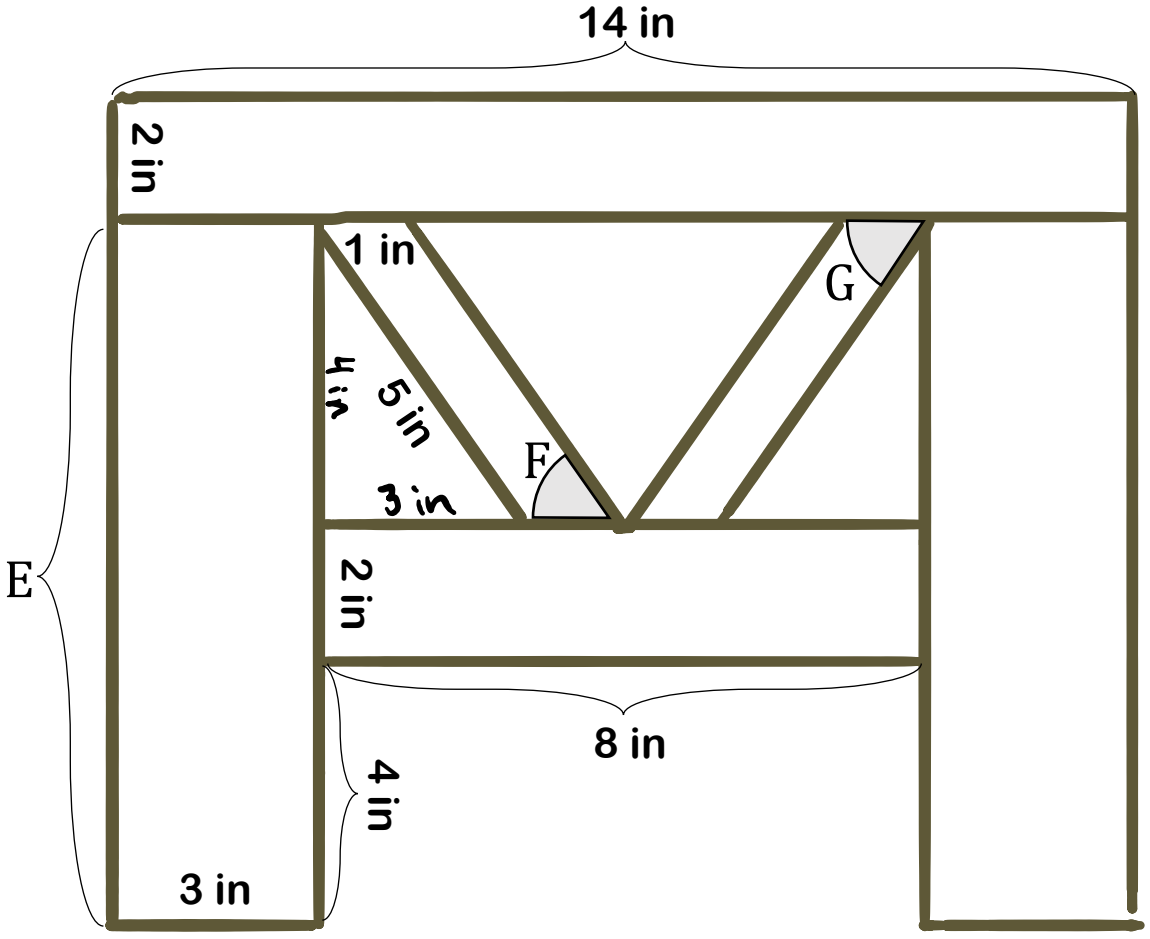
$\sin^{-1}\left(\frac{5}{13}\right) = 22.6°$

$90° - 22.6° =$
67.4

$12-2-2-3=5$
$5^2 + x^2 = 13^2$
$x = 12$
$12+2+2=16$

Real World Applications for Right Triangles
Practice Test

Find the missing measurements of this table:



E= 10 in

∡F= 53.1°

∡G= 53.1°

8/2=4 4-1=3in

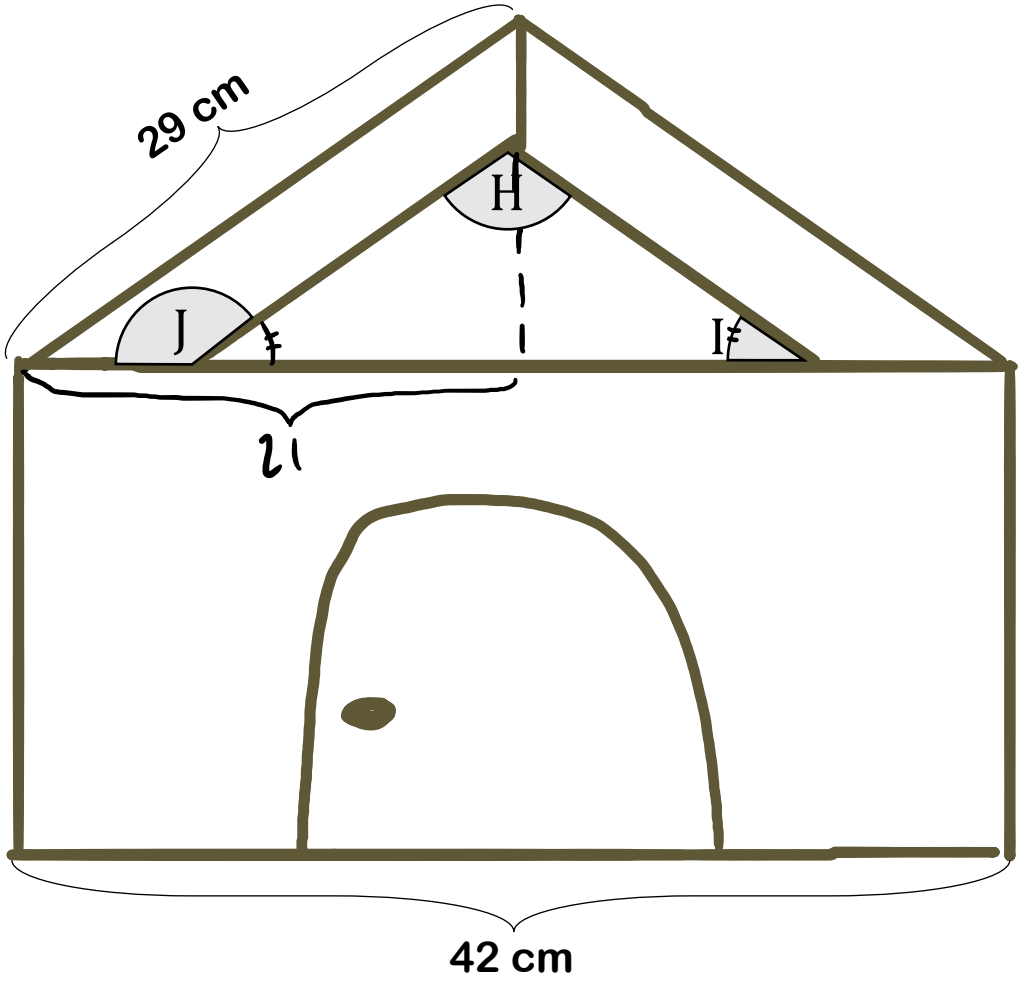
$3^2 + x^2 = 5^2$ x=4in

4+2+4=10 in

$\cos^{-1}(3/5) = 53.1°$

Real World Applications for Right Triangles
Practice Test

Find the missing measurements of this cabin:



∡J= 136.4°

∡H= 92.8°

∡I= 43.6°

42/2 = 21

$29^2 = 21^2 + x^2$

x = 20

$\sin^{-1}\left(\frac{21}{29}\right) = 46.4°$

46.4° × 2 = 92.8°

180-90-46.4=43.6°

180-43.6=
136.4°